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A USER'S GUIDE TO THE SUDAN COMPUTER PROGRAM FOR DETERMINING THE VIBRATION MODES OF STRUCTURAL SYSTEMS

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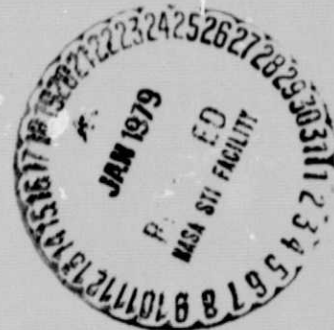
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By

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SUMMARY

The use of the SUDAN computer program for analyzing structural systems for their natural modes and frequencies of vibration is described. SUDAN is based on a substructures methodology and is intended for structures which can be represented as an equivalent system of beam, spring, and rigid-body substructures. A finite element stiffness technique in combination with a lumped mass method which allows for non-diagonal masses is used to model each of the substructures which comprise the system. User-written constraint equations are used to analytically join the mass and stiffness matrices of the substructures to form the mass and stiffness matrices of the complete structure from which all the frequencies and modes of the system are determined. A feature of the SUDAN program is the capability to treat the case in which both the mass and stiffness matrices of the coupled system may be singular simultaneously. A general description of the computer program is given, the computer hardware and software specifications are indicated, and the input required by the program is described. The usage of the program is illustrated by a simple example. The SUDAN program and related documentation can be obtained from the Computer Software Management and Information Center (COSMIC) at the University of Georgia.

INTRODUCTION

Two computational procedures for analyzing complex structural systems for their natural modes and frequencies of vibration using a substructures methodology are presented in reference 1. One of the procedures described is a direct method based on solving the eigenvalue problem associated with a finite-element representation of the complete structure. This computational procedure has been implemented in a special-purpose computer program designated SUDAN (Substructuring in Direct Analysis). The SUDAN program is intended for structures which can be represented as an equivalent system of beam, spring, and rigid-body substructures. A finite-element stiffness technique in combination with a lumped mass method which allows for non-diagonal masses is used to generate the mass and stiffness matrices for each of the substructures comprising the system. The mass and stiffness matrices of the individual substructures are joined analytically by means of user-written equations of constraint to form a finite-element model of the complete structure from which all the frequencies and modes of the system can be determined. A feature of the SUDAN program is the capability to treat the case in which both the mass and stiffness matrices of the coupled system may be singular simultaneously. The SUDAN computer program is somewhat limited in the maximum size of the problem which can be accommodated since it employs an in-core eigensolution technique. However, the program has been used quite successfully by the authors and their colleagues in several airframe vibration analyses carrying engineering responsibility. This experience has demonstrated a wide range of engineering applicability for the program.

The purpose of this report is to present a user's guide for the SUDAN program. A general description of the program is given, the computer hardware and software specifications are indicated, and the inputs required by the program are described. A sample case consisting of an assembly of beams configured in the shape of an airplane and supported by springs is provided to illustrate the manner of preparing input data and to indicate the format of the output. The SUDAN program and related documentation can be obtained from the Computer Software Management and Information Center (COSMIC) at the University of Georgia.

The present report relies on reference 1 for a description of the analytical basis of the computational procedures which are implemented in SUDAN and on reference 2 for a discussion of representing a structure as an assembly of beam, spring, and rigid-body substructures. For these reasons, references 1 and 2 and the present report should be regarded as companion reports.

GENERAL DESCRIPTION OF PROGRAM

The SUDAN program is a system comprised of two programs—DSTIFF and BJD5—which are executed sequentially with disk files providing the communication between them.

Organization

Based on a user-established idealization of a structure as an equivalent system of beam, spring, and rigid body substructures, Program DSTIFF employs the combined lumped-mass/finite element stiffness technique described in reference 2 to generate the stiffness and mass matrices for the structural block(s) comprising each of the constituent substructures. The uncoupled system stiffness and mass matrices, given by the composite matrices containing

the stiffness and mass matrices of the individual substructures as submatrices on the principal diagonals, are then formed. Spring and/or mass terms coupling either the blocks of a substructure submatrix or the substructure submatrices, as well as any other additional spring or mass terms, are then added to the uncoupled system stiffness and mass matrices. User-written constraint equations enforcing inter-substructure displacement compatibility are then applied according to the method of Walton and Steeves (ref. 3)* leading to the generalized eigenvalue problem equation

$$[K] \{X\} = \text{LAMBDA} [M] \{X\} \quad (1)$$

where $[K]$ is the coupled system stiffness matrix, $[M]$ is the coupled system mass matrix, LAMBDA is the eigenvalue which corresponds to the square of the natural frequency, and $\{X\}$ is the eigenvector which corresponds to the mode shape in terms of generalized coordinates arising from solution of the constraint equations.

Program BJD5 solves the generalized eigenvalue problem given by equation 1. This program was written by the second author and is based on an algorithm devised by Walton and Durling (unpublished work, 1966) to reduce the free vibration equations of motion to symmetric standard eigenvalue form in the general case when both the stiffness and mass matrices may be singular. The analytical basis of the BJD5 algorithm is reviewed in reference 1. The solution provides all the eigenvalues and the eigenvectors of equation 1. Since the solution eigenvectors are generalized mode shapes, they are transformed back to the physical coordinates of the original uncoupled system for interpretation by the user.

*See Appendix of reference 1 for description of an efficient computational procedure for implementing the method of Walton and Steeves.

Continuity between references 1 and 2 and the present report is facilitated by the use of the same notation whenever possible and through extensive use of comments in the source programs.

Computer Hardware and Software Specifications

The SUDAN programs—DSTIFF/BJD5— and their subroutines are written in FORTRAN IV. They have been compiled using the FORTRAN (FTN) compiler and executed on the Network Operating System (NOS 1.2) on Control Data Corporation (CDC) CYBER 173/175 digital computers at the Langley Research Center (LaRC) Computer Complex. These programs were developed on CDC 6400/6600 computers using the RUN compiler on a modified SCOPE 2.3 Operating SYSTEM (ICOPS) at the LaRC Computer Complex. The only program change required to convert the programs from ICOPS to NOS was one end-of-file test in Program DSTIFF. Library subroutines JACTV and MATINV are included in the source programs. The intrinsic functions ABS and MOD and the basic external function SQRT utilized by the programs and subroutines are presumed to be available on other digital computer systems. Experience gained by the second author in converting programs received from other sites indicates that only a few changes would be required to make these programs operational on other digital computers.

Compatibility features.— Some of the FORTRAN IV features employed or avoided in SUDAN in an effort to provide compatibility among various computers are as follows:

1. All variables and arrays are identified by a symbolic name consisting of from one to six letters or digits beginning with a letter.
2. All variables and arrays are typed implicitly by the first character of the name: I, J, K, L, M, and N imply type INTEGER, and the other letters

imply type REAL. No explicit type statements are used.

3. Only returns of the type RETURN are used in the subroutines.

When a RETURN statement is encountered control is transferred to the next statement following the CALL statement in the calling program.

4. There are no ENTRY statements in the subroutines.

5. Arrays have either one or two dimensions.

6. Only one 2-branch IF statement is used in SUDAN, and it is the statement containing the end-of-file test in Program DSTIFF.

7. Adjustable dimensions are formal arguments in all of the subroutines.

8. The H specification is used in FORMAT statements containing a string of alphanumeric or special characters including blanks rather than delimiting the string with special symbols.

9. Real constants formerly contained in arithmetic and relational expressions in a given routine have been replaced by real variables. These variables are not included in a blank COMMON or a labeled COMMON statement. In each routine these variables are assigned values in DATA statements and are not redefined during execution. Each constant in the DATA statements contains from two to fourteen digits, a decimal point, and an exponent comprised of the letter E followed by a signed or an unsigned integer constant. The three constants containing fourteen digits are in DATA statements in Subroutines JACTV and FREQ.

10. Hollerith constants use the H specification. The letter H is preceded by the unsigned integer 4 and is followed by four alphanumeric or special characters including blanks. In these programs, Hollerith constants are used only to define elements of the HMTX array. Since the HMTX array is included in a labeled COMMON statement, and since some computers require that a

BLOCK DATA subprogram be used to initialize arrays or variables in COMMON with DATA statements, DATA statements are not used to define elements of the HMTX array.

Conversion considerations. - It is recommended that the SUDAN programs and subroutines be converted to double precision if the computer to be used provides less than approximately fourteen digits in single precision computation. All real variables and arrays except PROBID and HMTX in Program BJD5 should be double precision. As a rule of thumb, when the difference of exponents (spread) between the largest eigenvalue and the smallest non-zero eigenvalue for a given eigenvalue problem approaches the number of digits of precision provided by the computer, the number of significant digits in the smallest eigenvalue may decrease to the point where even the first digit is not correct.

Some specific modifications and points to be considered in converting the SUDAN programs and subroutines for use on other than CDC computers are as follows:

1. Delete the PROGRAM statement in both Programs DSTIFF and BJD5 to make them main programs, and provide file assignments for the programs as required on a given computer.
2. The precision of real variables and real arrays in the routines can be extended to double precision by including the appropriate type statement for the given computer. The type statements may be DOUBLE PRECISION, IMPLICIT, or REAL with the extended length specification or combinations thereof. The precision of REAL constants can be increased by changing the letter E in the exponent of each constant to the letter D. REAL constants requiring increased precision occur only in DATA statements. It is noted that the

precision of two arrays—HMTX and PROBID—should not be extended since these arrays are used for Hollerith data. The arrays and the routines in which they are used are as follows:

HMTX - DSTIFF, WMTXC, BJD5, PMTXC, FREQ

PROBID - DSTIFF, BJD5

3. If extended precision is provided in the routines, the appropriate number of digits should be added to the 14-digit constants which are in DATA statements in Subroutines JACTV and FREQ.

4. A variable named ONEMIL is initialized to 10^6 in DATA statements in both DSTIFF and BJD5, and it is not redefined during execution. This variable is used in determining the number of positive, non-zero eigenvalues since "zero" eigenvalues may not be precisely zero in the computer. Experience has indicated that 10^6 is an appropriate value for this test on CDC computers, but the value may have to be changed on other computers.

5. The value range of a single precision (real) word on CDC computers is 10^{-293} to 10^{+322} . The values of two variables, RHO and R1, used in the programs are system dependent. The variable RHO in Subroutine JACTV specifies the accuracy requirement, and it is initialized to $.1E-13$ in the DATA statement. The variable R1 and its reciprocal R2 in Subroutine MATINV are used to keep the magnitude of the computational values within the range allowed on a given computer. The variable R1 is initialized in the DATA statement to 10^{100} and is not redefined during execution. A programmer at a given site should be able to provide appropriate values for both RHO and R1.

6. If extended precision is provided in the routines, change each occurrence of the intrinsic function ABS to DABS and of the external

function SQRT to DSQRT. The functions and routines in which they occur are listed with the numbers enclosed in parentheses indicating the number of occurrences of the function in the routine, as follows:

ABS - JACTV(2), FREQ(1), MATINV(10)

SQRT - JACTV(1), BJD5(1), FREQ(1)

7. Program DSTIFF contains a two-branch IF statement which includes an end-of-file test (EOF function). This statement, which is numbered 231, will have to be deleted on some systems and the appropriate end-of-file-test parameter inserted in the READ statement, numbered 250, which immediately precedes it.

8. Formatted input and output statements and the routines in which they occur are as follows:

READ f,list - DSTIFF

PRINT f - DSTIFF, BJD5

PRINT f,list - DSTIFF, BJD5, PMTXX, FREQ

WRITE(6,f)list - WMTXX

where f represents the FORMAT number, list represents the input/output (I/O) list to be transmitted, and 6 represents the output unit designator.

In the PROGRAM statements for DSTIFF and BJD5, TAPE5 is equivalenced to INPUT and TAPE6 is equivalenced to OUTPUT. Although READ and PRINT statements are known to be valid FORTRAN formatted I/O statements on two non-CDC computers, they may have to be changed before the programs can be used on some computers.

Problem size limitations. - The dimensions of the arrays in the source programs reflect a tailoring to the size of the sample problem. Larger problems can be accommodated by changing dimensions of the arrays and

the values of the variables set equal to the various dimensions and recompiling the programs. This is not a formidable task since all of the subroutines utilize variable dimensions and the program source decks contain detailed commentary. If the value of the variable MAXORD and dimensions associated with it are increased in Program DSTIFF, the user is cautioned to make appropriate changes in Program BJD5. By utilizing the maximum central memory (330K octal words) available to a job on the CDC CYBER 175 computers at the LaRC Computer Complex, the user can use approximately two hundred discrete coordinates to define the uncoupled system.

The primary reason for the problem size limitation is that the SUDAN programs use an in-core eigensolution technique—designated Subroutine JACTV. This subroutine incorporates the threshold variation version of the serial Jacobi algorithm. Since this method is particularly appropriate for the special requirements of the SUDAN programs, the user is cautioned against substituting another algorithm without first understanding the reasons for selection of JACTV as detailed in reference 1. If a replacement algorithm is used, it is recommended that comparison runs be made in both versions of the programs for a number of different problems to ensure that the integrity of the programs has been maintained.

General Input/Output Information

Before attempting to prepare data for a calculation, it is advisable to read through this report and also to be familiar with the substructure modeling techniques and the procedure for formulating the equations of constraint which are described in reference 2.

Only Program DSTIFF requires card input. Each of the programs produces both printed (formatted) and sequential binary (unformatted) output. The

binary output of Program DSTIFF is the binary input for Program BJD5. The binary output of Program BJD5 may be used, for example, as input to a user-selected program for generating and annotating mode-shape plots. In subsequent sections of this paper, the input and the output of the programs are described, and the card input for and the printed output from executions of a sample case are listed.

The card input data for Program DSTIFF must be prepared using the prescribed formats and must be assembled in the proper sequence. A consistent system of units must be used in preparing the applicable data. Integers must be input with correct values since many of the integers, or variables computed from them, are used to control the number of values read into arrays. The data described in the tables are designated either as required (REQ) or as conditional (COND). As implied by the designator, all required data must be prepared. If conditional data are included, the appropriate integer(s) must be input as indicated in the tabular description of these data. The number of cards required for each read statement is tabulated or can be computed from the expression which is tabulated in the column under "No. of Cards." Note that the quotient for each of these integer expressions must be incremented by one if the dividend is not an exact multiple of the divisor.

The identification card should contain alphanumeric information which will be meaningful to the user for later identification of the case. Integers must always be right justified in the specified fields. More latitude is allowed for the single-precision, floating-point numbers: a decimal point in the field overrides the decimal position specified in the format, and an exponent is not required if a properly positioned decimal point is included.

When an exponent is included, it must be right justified in the field with the letter E and/or the sign of the exponent preceding it.

As noted previously, the binary output of Program DSTIFF is the binary input for Program BJD5. The data are contained on two files—TAPE 3 and TAPE4. No file manipulation is required between executions when these programs are executed sequentially on the same job. When the programs are executed on separate job submittals, the user must save both TAPE 3 and TAPE 4 after the execution of Program DSTIFF on the first job, and then must retrieve these files on the second job prior to the execution of Program BJD5. The file names, TAPE3 and TAPE4, are declared, respectively, on the DSTIFF program statement as the third and fourth files and on the BJD5 program statement as the second and third files. In order to avoid errors on file assignments, the user is cautioned that Program BJD5 uses TAPE4 as a scratch file for temporary data storage after it reads the coupled mass and stiffness matrices from it. Program BJD5 performs no write operations on TAPE3.

The binary output from Program BJD5 is contained on TAPE9. If this file is needed for a later job submittal, it must be saved by the user prior to the termination of the job on which it is created. The file name TAPE9 is declared as the fifth file on the BJD5 program statement.

DESCRIPTION OF SUDAN INPUT AND OUTPUT

The SUDAN computer program system is comprised of two programs—DSTIFF and BJD5—and their subroutines. Usually these programs are executed sequentially with disk communication between them. Program DSTIFF calls subroutines JACTV, WMTXC, and ZEROM; Program BJD5 calls subroutines FREQ, JACTV, MATINV, and PMTXC. The subroutines are included in the SUDAN source

program. A sample deck structure for execution of the SUDAN programs on the CYBER 173/175 computers at the IARC Computer Complex is shown in figure 1.

This section is divided into four subsections:

1. Input data for Program DSTIFF.
2. Output for Program DSTIFF.
3. Input for Program BJD5.
4. Output for Program BJD5.

Input Data for Program DSTIFF

The data deck for Program DSTIFF may consist of as many as five different types of data sets identified and assembled as follows:

1. Initial data
2. Block data
3. Additional mass data
4. Additional stiffness data
5. Constraint data

Initial data, block data, and constraint data must always be prepared; the additional mass data and/or the additional stiffness data are included when appropriate for the case. Definition of a beam substructure requires at least one block of data and may require up to four blocks (bending in two principal directions, torsion, and extension), depending on the freedoms assigned to the beam component. It is noted, for clarity, that the data deck for Program DSTIFF is contained in one (1) record.

Initial data.- The initial data consist of two cards.

No. of Cards	Option	FORMAT	Description of Initial Data
1	REQ	20A4	PROBID = one card containing alphanumeric identification for the case (maximum of 80 characters)
1	REQ	6I4	NBLKS, NORDER, NCEQS, NMASAD, NSPRAD, IPRINT NBLKS = number of blocks in the uncoupled system stiffness and mass matrices NORDER = order of the uncoupled system NCEQS = number of constraint equations NMASAD = number of mass terms to be added <u>on or above</u> the diagonal of the uncoupled system mass matrix. The program will provide symmetry. NSPRAD = number of spring terms to be added <u>on or above</u> the diagonal of the uncoupled system stiffness matrix. The program will provide symmetry. IPRINT = control integer for printing output IPRINT = 1, standard output will be printed for the case IPRINT = 2, the assembled uncoupled stiffness and mass matrices as well as the standard output will be printed. The order of these two square matrices is NORDER. The additional printing may be helpful, especially to a new user, in determining that the conditional additional mass and/or stiffness terms have been input correctly.

The two initial data cards are followed by NBLKS sets of block data.

Block data.- There are NBLKS sets of block data. Each set provides the input data for generation of one block of the uncoupled system stiffness and mass matrices. The first card in each set contains five integers.

No. of Cards	Option	FORMAT	Input Variables with Description for Card 1 of a Block Data Set
1	REQ	5I4	<p>KK, LOC, MK, MM, ISYM</p> <p>KK = order of the block which is equal to the number of degrees of freedom of the block</p> <p>LOC = control integer for block type</p> <p>LOC = 1, beam bending</p> <p>LOC = 2, beam torsion</p> <p>LOC = 3, beam extension</p> <p>LOC = 4, spring or rigid body component</p> <p>MK = control integer for spring components</p> <p>MK = 1, read stiffness matrix from cards</p> <p>MK = 0, no read, block will be null</p> <p>MM = control integer for rigid body components</p> <p>MM = 1, read mass matrix from cards</p> <p>MM = 0, no read, block will be null</p> <p>ISYM = control integer indicating whether symmetry is used in formulating the problem</p> <p>ISYM = 1, the program will multiply the block stiffness and mass matrices by 2.0. This option is used only when a component is out of the plane of symmetry and the complete problem is being solved as a pair of symmetric and antisymmetric problems</p> <p>ISYM = 0, no multiplication - either not applicable to component or the complete problem is being solved without taking advantage of symmetry.</p>

The user completes preparation of a given set of block data from the table which describes the additional input variables required for the block type specified by the value of LOC. The data descriptions for both beam torsion (LOC = 2) and beam extension (LOC = 3) are included in the same table since they are similar.

No. of Cards	Option	FORMAT	Additional Input Variables with Description for Beam Bending, LOC = 1, Block Data
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1 REQ 3I4 K, N, KASE

The free-free stiffness matrix is generated in the partitioned form

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

where A is KxK, B is KxN, B^T is NxK, and C is NxN. For the free-free case, K = N = (KK)/2

K = order of matrix A

N = order of matrix C

KASE = control integer for mass matrix

KASE = 1, diagonal mass only

KASE = 2, diagonal mass and rotary inertia

KASE = 3, non-diagonal mass and rotary inertia.

The off-diagonal terms for this block must be input under the NMASAD option.

K/5	REQ	5E14.8	X array of K values, the running local coordinate positions (dimensional)
(K-1)/5	REQ	5E14.8	S array of K-1 values, the average stiffness, EI, between a pair of sequential coordinates
K/5	REQ	5E14.8	DM array of K values, lumped masses located at each of the K coordinates (zero values are admissible)
N/5	COND	5E14.8	RI array of N values, lumped rotary inertia for each of the N rotational coordinates (zero values are admissible). These values must be input if KASE is not equal to 1.

If the beam bending data do not complete the NBLKS-th set of block data, the user must turn back to the beginning of the block data section to start the next set of block data for the case. If, however, the set number for beam bending data is equal to NBLKS, data preparation is continued with the next type of data which may be conditional additional mass data, conditional additional stiffness data, or constraint data, in that order.

No. of Cards	Option	FORMAT	Additional Input Variables with Description for Beam Torsion, LOC = 2, and for Beam Extension, LOC = 3, Block Data
1	REQ	2I4	K, KASE K = order of free-free stiffness matrix and beam torsional inertia or extensional mass matrix KASE = identification integer for mass matrix KASE = 4, diagonal torsional inertia or extensional mass KASE = 5, non-diagonal torsional inertia or extensional mass. The off- diagonal terms for this block must be input under the NMSAD option.
K/5	REQ	5E14.8	X array of K values, the running local coordinate position (dimensional)
(K-1)/5	REQ	5E14.8	S array of K-1 values S array = GJ for beam torsion (LOC=2) S array = AE for beam extension (LOC=3)
K/5	REQ	5E14.8	RI array of K values (zero values are admissible) RI array = lumped torsional inertias for beam torsion (LOC=2) RI array = lumped extensional masses for beam extension (LOC=3)

If the beam torsion or beam extension data do not complete the NBLKS-th set of block data, the user must turn back to the beginning of the block data section to start the next set of block data for the case. If, however, the set number for beam torsion or beam extension data is equal to NBLKS, data preparation is continued with the next type of data which may be conditional additional mass data, conditional additional stiffness data, or constraint data, in that order.

No. of Cards	Option	FORMAT	Additional Input Variables with Description for Spring or Rigid-Body Components, LOC = 4, Block Data
(KK) ² /5	COND	5E14.8	A array of KK x KK values, the stiffness matrix for a spring component. It is read in continuously by rows if the value of MK for the block is not zero. Each card has five values except the last card which may have blank fields to the right of the last value. The program provides a null mass matrix for a spring component.
(KK) ² /5	COND	5E14.8	B array of KK x KK values, the mass matrix for a rigid-body component. It is read in continuously by rows if the value of MM for the block is not zero. Each card has five values except the last card which may have blank fields to the right of the last value. The program provides a null stiffness matrix for a rigid-body component. -- NOTE -- The case in which the rigid body has a null mass matrix is admissible. This option is selected by setting both MK and MM to zero.

If the spring or the rigid-body component data do not complete the NBLKS-th set of block data, the user must turn back to the beginning of the block data section to start the next set of block data for the case. If, however, the set number for the spring or the rigid body component data is equal to NBLKS, data preparation is continued with the next type of data which may be conditional additional mass data, conditional additional stiffness data, or constraint data, in that order.

Additional mass data.- If the centers of gravity of the lumped masses for a beam substructure are not located on the elastic axis there will be mass terms coupling the blocks in the mass matrix corresponding to the beam substructure. Additional mass terms (diagonal, off-diagonal, and coupling) will also arise if the mass matrix of a rigid body component not treated as a substructure is combined with the mass matrix of the beam component to which it is attached. The use of consistent mass matrices for bending, torsion, or extension will lead to non-diagonal mass matrices for those blocks. The user is referred to reference 2 for discussion of these aspects. The control integer NMSAD determines whether additional mass data are to be read. All off-diagonal mass data for beam components are input as additional mass data. The row number and the column number provided with each of the NMSAD terms are the row and the column of the element in the uncoupled system mass matrix to which the additional term is to be added. The program will provide symmetry, so data are input only for values which appear on or above the diagonal.

CAUTION: For a symmetric or an anti-symmetric formulation of a problem, the masses to be added to the uncoupled system mass matrix must be multiplied by 2.0 by the user prior to input if they correspond to components out of the plane of symmetry.

No. of Cards	Option	FORMAT	Input Variables with Description for Additional Mass Data
NMSAD/4	COND	4(2I3,E14.8)	Row number, column number, and value for each of the NMSAD additional mass terms to be added. Blank fields may not be present in any except the last card, and then may be only those fields to the right of the NMSAD-th group of three numbers. Note caution in the previous paragraph.

Data preparation is continued with conditional additional stiffness data if NSPRAD is not zero and with constraint data if NSPRAD equals zero.

Additional stiffness data. - Spring terms which couple the substructure submatrices will arise if the stiffness of springs having neither end tied to ground are combined with the stiffness matrices of the components to which they are attached. For springs which have one end tied to ground, additional spring terms (diagonal, off diagonal, and coupling) will also arise if the other end of the spring is attached to a beam or rigid body component at a point other than a station established as a degree of freedom in the problem formulation. The user is referred to reference 2 for discussion of these aspects. The control integer NSPRAD determines whether additional stiffness data are to be read. The row number and the column number provided with each of the NSPRAD terms are the row and the column of the element in the uncoupled system stiffness matrix to which the additional term is to be added. The program will provide symmetry, so data are input only for values which appear on or above the diagonal.

CAUTION: For a symmetric or an anti-symmetric formulation of a problem, the spring terms to be added to the uncoupled system stiffness matrix must be multiplied by 2.0 by the user prior to input if they correspond to components out of the plane of symmetry.

No. of Cards	Option	FORMAT	Input Variables with Description for Additional Stiffness Data
NSPRAD/4	COND	4(2I3,E14.8)	Row number, column number, and value for each of the NSPRAD additional spring terms to be added. Blank fields may not be present in any except the last card, and then may be only those fields to the right of the NSPRAD-th group of three numbers. Note caution in the previous paragraph.

Data preparation is continued with constraint data.

Constraint data. - Since all elements of the constraint matrix are set to zero before constraint data are read, only non-zero elements are input. The number of rows in the constraint matrix is equal to the number of constraint equations, NCEQS. The number of columns in the constraint matrix is equal to the order of the uncoupled system, NORDER. In the description which follows, NZECM is used only to indicate the total number of non-zero elements of the constraint matrix and is neither an input parameter nor a program variable.

No. of Cards	Option	FORMAT	Input Variables with Description for Constraint Data
NZECM/4	REQ	4(2I3,E14.8)	Row number, column number, and coefficient for each non-zero value in the constraint matrix. Blank fields may not be present in any except the last card, and then may only be those fields to the right of the last (NZECM-th) group of three numbers. The maximum value for a row number is NCEQS and for a column number is NORDER.

Output for Program DSTIFF

Printed output. - The printed output for Program DSTIFF consists of a block by block summary of the input data arranged in tabular form and the eigenvalues of the constraint eigenvalue problem which is associated with the method of Walton and Steeves (ref. 3) for imposing the equations of constraint. An option is provided for printing the uncoupled system mass and stiffness matrices. This additional output may be helpful in identifying input errors in the placement of the additional mass and stiffness elements in the uncoupled system mass and stiffness matrices. The printed sample output presented in a subsequent section does not include this optional output.

Binary output. - The binary output of Program DSTIFF is not described here since it is fully described as the binary input for Program BJD5.

Input for Program BJD5

Card input.- There is no card input for Program BJD5.

Binary input.- The binary input for Program BJD5 is the binary output from Program DSTIFF, and it is contained on two files, TAPE3 and TAPE4. This input includes problem identification, numbers which specify the order of matrices contained on the files, the elements of the connectivity matrix (BETA) which enforces displacement compatibility at the interfaces of the substructures, and the elements of the coupled system mass and stiffness matrices. The BETA matrix is used at the end of Program BJD5 to transform the modal matrix back to the original physical coordinates of the uncoupled system to facilitate interpretation by the user.

File Name	Record Number	Description of the Binary Input for Program BJD5
TAPE3	1	PROBID, NRBETA, NCBETA PROBID = the card image (80 characters) of the problem identification card provided as input for Program DSTIFF NRBETA = number of rows in the BETA matrix NCBETA = number of columns in the BETA matrix and order of both the coupled system mass matrix and the coupled system stiffness matrix
	2 through (NRBETA + 1)	NRBETA records each of which contains NCBETA elements for a row of the BETA matrix
TAPE4	1	This record contains all the elements of the coupled system mass matrix of order NCBETA.
	2	This record contains all the elements of the coupled system stiffness matrix of order NCBETA.

Output for Program BJD5

Printed output. - The printed output for Program BJD5 consists of the eigenvalues of the coupled system mass matrix, the eigenvalues and frequencies of the coupled system, and the eigenvectors of the coupled system transformed back to the original physical coordinates of the uncoupled system.

Binary output.- The binary output of Program BJD5 is contained on one file, TAPE9, and includes the number of rows and columns in the modal matrix, problem identification, frequencies, and vectors of the modal matrix which have been transformed back to the original physical coordinates of the uncoupled system. These data may be used, for example, in a program selected by the user to generate and annotate mode-shape plots. If this output is required for a later job submittal, it must be saved either on a magnetic tape or on a permanent file prior to termination of the job on which it is generated.

CAUTION: In order to provide compatibility among various computers, the PROBID array contains the 80 characters of the problem identification card in 20 words each of which has 4 left-justified characters. When the PROBID array is to be used to annotate the plot(s) generated by a user-supplied plotting program, the user is reminded that the 80 characters are contiguous only if the computer word has four 8-bit bytes (32 bits). If, for example, the computer word has 36 bits or 60 bits, several suggestions to assist the user are presented, as follows:

1. Plot four characters at a time from each of the 20 words in the array.
2. Reformat the 80 characters in memory from the 20-word array to a compatible array for the given computer. For example, assuming PROBID is dimensioned 20 and CASE is dimensioned 8, the transfer could be accomplished

on CDC computers by including the following two statements in the program:

```
ENCODE(80,5000,CASE) PROBID
```

```
5000 FORMAT(20A4)
```

Since the word size on CDC computers is 60 bits, the eight words in the CASE array will contain the 80 contiguous characters after execution of the ENCODE statement.

3. Modify Program BJD5 so that the 80 characters are reformatted into contiguous locations in an array as described in the second suggestion, and then write the new array instead of PROBID on TAPE9.

4. Perform a binary (unformatted) read of only NRB and K from the first record on TAPE9, and provide an input problem identification data card to be read by a formatted read statement which will place the 80 characters in contiguous locations in an array on the given computer. On CDC computers, the format would be 8A10, and the array would be dimensioned 8.

File Name	Record Number	Description of the Binary Output from Program BJD5
TAPE9	1	NRB, K, PROBID NRB = number of rows in the modal matrix after it has been transformed back to the original physical coordinates of the uncoupled system K = number of frequencies and also number of columns in the modal matrix PROBID = the card image of the problem identification card provided as input for Program DSTIFF. It is noted that the 80 characters are contained in 20 words each of which has 4 left-justified characters. See caution which precedes this table.
	2	K calculated frequencies (Hz) of the coupled system arranged in ascending order.
	3 through (K + 2)	K records each of which contains the NRB elements of a vector (column) of the modal matrix after it has been transformed back to the original physical coordinates of the uncoupled system. The k-th vector is associated with the k-th frequency.

SAMPLE CASE TO ILLUSTRATE USAGE OF PROGRAM

Description of Configuration

The SUDAN program was applied to the vibration analysis of a model consisting of an assembly of beams configured in the shape of an airplane in reference 1. This model is the basis of the sample case selected to illustrate the usage of the program. The geometric properties of the model are given in figure 2. Since attention will be directed to a symmetric vibration analysis as in reference 1, advantage can be taken of symmetry and only one half of the model need be explicitly considered. Thus, for analysis purposes, the mathematical model can be considered to consist of three substructures: fuselage, wing, and tail. For symmetric motions, the component motions of interest are vertical bending of the fuselage beam and vertical bending and torsion of the wing and tail beams. The distributed mass of the fuselage, wing, and tail beams is lumped at discrete points along the elastic axes of the respective members such that the sectional mass, mass unbalance about the elastic axis, and the torsional inertia are preserved. Although the rotary inertia in bending associated with each of these lumped masses is not zero, it is assumed unimportant and taken to be zero. Since the mass axis and elastic axis of each beam comprising the model are coincident, the static unbalance of the wing and tail masses about their respective elastic axes is zero. However, to illustrate the use of the NMSAD option in the program, the static unbalances of the wing masses about the elastic axis are assumed to be non-zero. This will inertially couple wing bending with wing torsion. A sketch of the resulting lumped-mass model of the beam assembly is given in figure 3. The fuselage and wing support springs shown in figure 3 were not part of the configuration analyzed in reference 1 but are introduced here to illustrate

the use of the NSPRAD option in the program. The fuselage support springs consist of a translational spring having a spring rate k_1 and a rotational spring having spring rate k_2 which are attached to the elastic axis at the seventh fuselage station. The wing support springs are translational springs having a spring rate k_3 and are attached to the leading edge of the wing (2.54 cm from elastic axis) at the local axis station corresponding to the fourth wing station. Since these springs are attached to the wing off the the elastic axis, additional coupling of wing bending and wing torsion will occur through stiffness terms associated with the wing support springs.

Each fuselage station has two degrees of freedom: vertical translation and rotation. Each wing and tail station has three degrees of freedom: vertical translation, rotation, and torsion. These discrete degrees of freedom are identified and ordered in Table I. A total of 65 discrete degrees of freedom are associated with the uncoupled system, 26 of which correspond to coordinates with no mass as a consequence of assuming the rotary inertias to be zero. The model structural properties, as discretized for the analysis in accordance with the lumped-mass model shown in figure 3, are summarized in Table II.* The procedure for determining the numerical values of the stiffness coupling terms associated with the support springs and for identifying the position of the mass static unbalance terms and spring terms in the substructure mass and stiffness matrices is described in reference 1. Both the wing mass static unbalance terms and the stiffnesses of the support springs shown in Table II were deliberately chosen to be small so as to represent only small perturbations from the configuration treated in reference 1, thereby

*It should be noted that the structural properties of the wing and tail beams are based on cuts perpendicular to the elastic axes of those components.

facilitating a comparison with the results shown therein if desired. A summary of the constraint equations required to analytically join the substructures is given in Table III.

Listings of the input and printed output for the sample case are given in the next two sections, respectively. The standard output option (IPRINT = 1) was selected for the sample case and thus the optional output (the uncoupled system mass and stiffness matrices) is not included in the sample output listing.

The sample case considered has no unrestrained (i.e., rigid-body) degrees of freedom, and thus none of the eigenvalues LAMBDA are zero. If a structure is unrestrained, the eigenvalues corresponding to the rigid-body modes should be zero. In practice, however, the eigenvalues which are calculated for the rigid-body modes may not be exactly zero. Because some of these "zero" eigenvalues may be negative, Program BJD5 takes the square root of the absolute value of LAMBDA to obtain the natural frequency in radians per second.

AIRPLANE BEAM ASSEMBLY - SAMPLE CASE FOR SUDAN PROGRAM

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Sample Printed Output for SUDAN

AIRPLANE BEAM ASSEMBLY - SAMPLE CASE FOR SUDAN PROGRAM

P R O B L E M I D

***** OUTPUT FOR PROGRAM DSTIFF *****

NUMBER BLOCKS IN UNCOUPLED SYSTEM MASS AND STIFFNESS MATRICES = 5

ORDER OF UNCOUPLED SYSTEM = 65

NUMBER OF CONSTRAINT EQUATIONS = 6

NUMBER OF MASS TERMS TO BE ADDED = 7

NUMBER OF SPRING TERMS TO BE ADDED = 5

PRINT OPTION = 1

*
* BLOCK NUMBER 1 *

BEAM BENDING

KK	LOC	MK	MM	ISYM	K	N	KASE
26	1	0	0	0	13	13	1

J	X(J)	EI(J)	MASS(J)	RI(J)
1	0.	.13715000E+03	.36480000E-01	0.
2	.10160000E+00	.13715000E+03	.72960000E-01	0.
3	.20320000E+00	.13715000E+03	.72960000E-01	0.
4	.30480000E+00	.13715000E+03	.72960000E-01	0.
5	.40640000E+00	.13715000E+03	.65670000E-01	0.
6	.48770000E+00	.13715000E+03	.52653000E+00	0.
7	.60960000E+00	.13715000E+03	.80260000E-01	0.
8	.71120000E+00	.13715000E+03	.72960000E-01	0.
9	.81280000E+00	.13715000E+03	.72960000E-01	0.
10	.91440000E+00	.13715000E+03	.72960000E-01	0.
11	.10160000E+01	.13715000E+03	.65670000E-01	0.
12	.10973000E+01	.13715000E+03	.72960000E-01	0.
13	.12192000E+01	.13715000E+03	.43780000E-01	0.

*
* BLOCK NUMBER 2 *
*

BEAM BENDING

KK	LOC	MX	MY	ISYM	K	N	KASE
14	1	0	0	1	7	7	1

J	X(J)	EI(J)	MASS(J)	RI(J)
1	0.	.27091000E+02	.64870000E-01	0.
2	.10160000E+00	.27091000E+02	.12975000E+00	0.
3	.20320000E+00	.27091000E+02	.12975000E+00	0.
4	.30480000E+00	.27091000E+02	.12975000E+00	0.
5	.40640000E+00	.27091000E+02	.12975000E+00	0.
6	.50800000E+00	.27091000E+02	.12975000E+00	0.
7	.60960000E+00	.27091000E+02	.64870000E-01	0.

*
* BLOCK NUMBER 3 *
*

BEAM TORSION

KK	LOC	MX	MY	ISYM	K	KASE
7	2	0	0	1	7	4

J	X(J)	GJ(J)	RI(J)
1	0.	.45110000E+02	.13950000E-04
2	.10160000E+00	.45110000E+02	.27900000E-04
3	.20320000E+00	.45110000E+02	.27900000E-04
4	.30480000E+00	.45110000E+02	.27900000E-04
5	.40640000E+00	.45110000E+02	.27900000E-04
6	.50800000E+00	.45110000E+02	.27900000E-04
7	.60960000E+00	.45110000E+02	.13950000E-04

```

*****
* BLOCK NUMBER 4 *
*****

```

BEAM BENDING

KK	LDC	MM	ISYM	K	N	KASE
12	1	0	0	1	6	1

J	X(J)	EI(J)	MASS(J)	RI(J)
1	0.	.27091000E+02	.32430000E-01	0.
2	.50800000E-01	.27091000E+02	.64870000E-01	0.
3	.10160000E+00	.27091000E+02	.64870000E-01	0.
4	.15240000E+00	.27091000E+02	.64870000E-01	0.
5	.20320000E+00	.27091000E+02	.64870000E-01	0.
6	.25400000E+00	.27091000E+02	.32430000E-01	0.

```

*****
* BLOCK NUMBER 5 *
*****

```

BEAM TORSION

KK	LDC	MM	ISYM	K	N	KASE
6	2	0	0	1	6	4

J	X(J)	GJ(J)	RI(J)
1	0.	.45110000E+02	.69800000E-05
2	.50800000E-01	.45110000E+02	.13940000E-04
3	.10160000E+00	.45110000E+02	.13940000E-04
4	.15240000E+00	.45110000E+02	.13940000E-04
5	.20320000E+00	.45110000E+02	.13940000E-04
6	.25400000E+00	.45110000E+02	.69800000E-05

**** MASS TERMS ADDED ****

J	ROW	COL	MASS(IJ)
1	27	41	.88960000E-04
2	28	42	.17742000E-03
3	29	43	.26690000E-03
4	30	44	.35586000E-03
5	31	45	.44482000E-03
6	32	46	.53378000E-03
7	33	47	-.26690000E-03

**** SPRING TERMS ADDED ****

J	ROW	COL	SPRING(IJ)
1	7	7	.35000000E+03
2	20	20	.56500000E-01
3	30	30	.17520000E+03
4	44	44	.11300000E+00
5	30	44	.44500000E+01

**** NON-ZERO ELEMENTS OF CONSTRAINT MATRIX ****

ROW	COL	COEFFICIENT	ROW	COL	COEFFICIENT	ROW	COL	COEFFICIENT
1	27	.10000000E+01	1	6	-.10000000E+01	2	34	.8602540E+00
3	34	-.50000000E+00	3	41	.2602540E+00	3	19	.10000000E+01
4	12	-.10000000E+01	5	54	.8602540E+00	5	60	.50000000E+00
6	60	.8602540E+00	6	25	.10000000E+01	6	54	-.50000000E+00
						2	41	.50000000E+00
						4	48	.10000000E+01
						6	54	-.50000000E+00

100	100	100	100	100	100
90	90	90	90	90	90
80	80	80	80	80	80
70	70	70	70	70	70
60	60	60	60	60	60
50	50	50	50	50	50
40	40	40	40	40	40
30	30	30	30	30	30
20	20	20	20	20	20
10	10	10	10	10	10
0	0	0	0	0	0

11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

***** OUTPUT FOR PROGRAM BIDS *****

EIGENVALUES OF COUPLED MASS MATRIX

1	+.2813300E+00
2	+.5950110E+00
3	+.5950076E+00
4	+.5950049E+00
5	+.5950027E+00
6	+.5950012E+00
7	+.1297455E+00
8	+.1297400E+00
9	+.1297400E+00
10	+.1297400E+00
11	+.1297400E+00
12	+.1297400E+00
13	+.1297400E+00
14	+.1297400E+00
15	+.1297400E+00
16	+.1297400E+00
17	+.1297400E+00
18	+.1297400E+00
19	+.1297400E+00
20	+.1297400E+00
21	+.1297400E+00
22	+.1297400E+00
23	+.1297400E+00
24	+.1297400E+00
25	+.1297400E+00
26	+.1297400E+00
27	+.1297400E+00
28	+.1297400E+00
29	+.1297400E+00
30	+.1297400E+00
31	+.1297400E+00
32	+.1297400E+00
33	+.1297400E+00
34	+.1297400E+00
35	+.1297400E+00
36	+.1297400E+00
37	+.1297400E+00
38	+.1297400E+00
39	+.1297400E+00
40	+.1297400E+00
41	+.1297400E+00
42	+.1297400E+00
43	+.1297400E+00
44	+.1297400E+00
45	+.1297400E+00
46	+.1297400E+00
47	+.1297400E+00
48	+.1297400E+00
49	+.1297400E+00
50	+.1297400E+00
51	+.1297400E+00
52	+.1297400E+00
53	+.1297400E+00
54	+.1297400E+00
55	+.1297400E+00
56	+.1297400E+00
57	+.1297400E+00
58	+.1297400E+00
59	+.1297400E+00

***** NUMBER OF FINITE MASS EIGENVALUES = 37 *****

F R E Q U E N C I E S A I R P L A N E R E A M A S S E M B L Y - S A M P L E C A S E F O R S U D A N P R O G R A M

J	LAMBDA(J)	OMEGA(J)	FREQ.(J), HZ		
1	.11155599629022E+10	.33399999444644E+05	.53157750108816E+04	←	Largest eigenvalue
2	.54038985668694E+09	.23246286944089E+05	.36997614756844E+04		(last mode)
3	.24791606385538E+09	.15745350547237E+05	.25059503703075E+04		
4	.2008001081334E+09	.14144964150303E+05	.22512409643783E+04		
5	.12955613066492E+09	.11382272649384E+05	.18115449557692E+04		
6	.12484342439929E+09	.1117335419618E+05	.17782915628560E+04		
7	.8206826270244E+08	.90591535312326E+04	.14418090647241E+04		
8	.72260802472292E+08	.85006354157964E+04	.13529181458460E+04		
9	.63568215941282E+08	.79729678251754E+04	.12689372404893E+04		
10	.62037154679312E+08	.7876368451458E+04	.12535627170101E+04		
11	.54724290430201E+08	.73975867977470E+04	.11773625058128E+04		
12	.51461071481612E+08	.71736372560656E+04	.11417198292510E+04		
13	.42337864067001E+08	.6506755241083E+04	.10355822637720E+04		
14	.40375564566021E+08	.63541769385201E+04	.10112986690460E+04		
15	.33561715604295E+08	.57932474143865E+04	.92202396255395E+03		
16	.23661407576016E+08	.4864292893135E+04	.77417727657264E+03		
17	.21374369969005E+08	.46232423653757E+04	.73581187556142E+03		
18	.15226291360796E+08	.39020880770167E+04	.62103660583716E+03		
19	.10093594930640E+08	.31770418522015E+04	.50564191518770E+03		
20	.90555361166217E+07	.30092417843406E+04	.47893570493648E+03		
21	.78547494836830E+07	.28026325987691E+04	.44605283176458E+03		
22	.62904215483255E+07	.25080712805512E+04	.39617194192654E+03		
23	.54818775433902E+07	.23413409711937E+04	.37263598902906E+03		
24	.46151755213060E+07	.21482959575687E+04	.34191192087140E+03		
25	.28781672306245E+07	.16965162040560E+04	.27000893991101E+03		
26	.20772898570923E+07	.14412806309294E+04	.22938693679502E+03		
27	.18860072466780E+07	.13733197903904E+04	.21857063308657E+03		
28	.11880709467577E+07	.10899866727431E+04	.17347676687136E+03		
29	.93645167192698E+06	.96770433084025E+03	.15401492770466E+03		
30	.52166631032392E+06	.72226470931641E+03	.11495199870854E+03		
31	.2809990233830E+06	.53009341000271E+03	.84366986502370E+02		
32	.10644262183542E+06	.32625545487457E+03	.51925168353983E+02		
33	.83355474394256E+05	.28871348149031E+03	.45950177716453E+02		
34	.18404069619398E+05	.13566159964927E+03	.21591214171934E+02		
35	.21860412540735E+04	.46755120084045E+02	.74413084762309E+01		
36	.16361961983367E+03	.12791388502961E+02	.20358127092550E+01		
37	.19779622116578E+00	.44474287084311E+00	.70783026299562E-01	←	Smallest eigenvalue
					(first mode)

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OF POOR QUALITY**

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[illegible]

CDL	33	34	35	36	37
1	4173500	27043501	61372500	99415000	100950001
2	4787000	12540000	55072500	866173000	892420000
3	5013500	109315001	49221500	805278000	744605000
4	15771000	500000000	49999100	740531000	537352000
5	22877000	100400002	33682500	676118000	359725000
6	26020000	33776000	20063100	626016000	216701000
7	30080000	580500000	28115000	555406000	463749000
8	60080000	650600000	26782500	458003000	175740000
9	75020000	604000000	28001000	473397000	335070000
10	73602000	494000000	27600000	379840000	320064000
11	62880000	201250000	36687000	329170000	705603000
12	97672000	401235001	36072500	277518000	847837000
13	115940000	650761000	36272500	209720000	100827000
14	954701000	630077001	71805000	619867000	174761301
15	985400000	630670000	71805000	619867000	174761301
16	800770000	630670000	70945000	618627000	174761301
17	707600000	544100001	67762000	611249200	174760201
18	631600000	333160001	65850000	605330000	174760201
19	666810000	155330000	65850000	550270000	174760201
20	13157000	817013001	24537100	502076000	174740001
21	170100000	817013001	11347000	502076000	174740001
22	832300000	12181000	20200000	570060000	174770001
23	445550000	208747001	35499000	562915000	174781001
24	176530000	208747001	46949000	558507000	174784001
25	283390000	335660001	48573000	556780000	174784001
26	281400000	335660001	48573000	556670000	174784001
27	255280000	337760000	45785000	588839000	126761000
28	336170000	4199005000	20631000	570511000	128937000
29	605002000	449743000	35119000	533735000	402660000
30	100740000	322133000	33510300	518480000	893682200
31	654470000	708700001	62060000	494114000	113700000
32	158007000	269910000	17683000	470001000	226633500
33	133051000	681470000	17446000	302607000	184266000
34	335400000	140000000	31420000	256302000	873797500
35	554860000	716770000	19772000	215560000	872592700
36	503050000	581270000	29030000	258600000	872513800
37	454060000	100111001	38850000	258600000	872361300
38	571660000	230110001	39650000	237080000	872361300
39	109735000	360720001	31743000	237080000	872372500
40	277180000	203720001	31743000	237080000	872372500
41	378400000	203720001	31743000	237080000	872372500
42	360500000	203720001	31743000	237080000	872372500
43	406011000	252770001	55072500	216220000	113362000
44	406011000	252770001	55072500	216220000	113362000
45	623030000	264820001	55072500	216220000	113362000
46	623030000	264820001	55072500	216220000	113362000
47	109272000	264820001	55072500	216220000	113362000
48	109272000	264820001	55072500	216220000	113362000
49	237420000	120720000	36687000	253118000	867637000
50	237420000	120720000	36687000	253118000	867637000
51	237420000	120720000	36687000	253118000	867637000
52	237420000	120720000	36687000	253118000	867637000
53	100200000	618620000	45017000	278000000	1069945000
54	116900000	106932001	27662000	278000000	873924400
55	381230000	209370001	31820000	276007000	873924400
56	356000000	209370001	31820000	276007000	873924400
57	450120000	209370001	31820000	276007000	873924400
58	450120000	209370001	31820000	276007000	873924400
59	625330000	274210000	37403000	2735635000	873924400
60	455520000	274210000	37403000	2735635000	873924400
61	470170000	274210000	37403000	2735635000	873924400
62	815900000	274210000	37403000	2735635000	873924400
63	278760000	274210000	37403000	2735635000	873924400
64	490600000	274210000	37403000	2735635000	873924400
65	490600000	274210000	37403000	2735635000	873924400

Fuselage beam	Vertical bending	Wing beam	Tail beam
Displacements	Slopes	Diplacements	Slopes
Twists	Twists	Slopes	Twists
Displacements	Slopes	Displacements	Slopes
Twists	Twists	Slopes	Twists
Displacements	Slopes	Displacements	Slopes
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ORIGINAL PAGE
OF POOR QUALITY

Fifth mode Fourth mode Third mode Second mode First mode (corresponds to smallest eigenvalue)

CONCLUDING REMARKS

A user's guide to the SUDAN computer program for analyzing structural systems for their natural modes and frequencies of vibration has been given. SUDAN is based on a substructures methodology and is intended for structures which can be represented as an equivalent system of beam, spring, and rigid-body substructures. A finite-element stiffness technique in combination with a lumped mass method which allows for non-diagonal masses is used to model each of the substructures which comprise the system. User-written constraint equations are used to analytically join the mass and stiffness matrices of the substructures to form the mass and stiffness matrices of the complete structure from which all the frequencies and modes of the system are determined. A general description of the program was given, the computer hardware and software specifications were indicated, and the input required by the program was described. The usage of the program was illustrated by a simple example.

REFERENCES

1. Kvaternik, R. G.: Development and Applications of Two Computational Procedures for Determining the Vibration Modes of Structural Systems. NASA TN D-7995, November 1975.
2. Kvaternik, R. G.: Considerations for the Application of Finite-Element Beam Modeling to Vibration Analysis of Flight Vehicle Structures. NASA TM X-73980, November 1976.
3. Walton, W. C., Jr.; and Steeves, E. C.: A New Matrix Theorem and Its Application for Establishing Independent Coordinates for Complex Dynamical Systems with Constraints. NASA TR R-326, October 1969.

TABLE I. - IDENTIFICATION OF DEGREES OF FREEDOM FOR SYMMETRIC VIBRATION
ANALYSIS OF AIRPLANE BEAM ASSEMBLY

FUSELAGE

$q_1 - q_{13}$	Displacements	}	Vertical bending
$q_{14} - q_{26}$	Slopes		

WING

$q_{27} - q_{33}$	Displacements	}	Vertical bending
$q_{34} - q_{40}$	Slopes		
$q_{41} - q_{47}$	Twists	}	Torsion

TAIL

$q_{48} - q_{53}$	Displacements	}	Vertical bending
$q_{54} - q_{59}$	Slopes		
$q_{60} - q_{65}$	Twists	}	Torsion

TABLE II.- DISCRETIZATION EMPLOYED FOR AIRPLANE BEAM ASSEMBLY

Station	Local Coordinate Position	Mass ^a	Torsional Inertia	EI	GJ	Mass Static Unbalance
	m	kg	kg-m ²	N-m ²	N-m ²	kg-m
Fuselage						
1	0.0000	0.03648	Not applicable ↓	137.15	Not applicable ↓	0.0
2	0.1016	0.07296		137.15		0.0
3	0.2032	0.07296		137.15		0.0
4	0.3048	0.07296		137.15		0.0
5	0.4064	0.06567		137.15		0.0
6 ^b	0.4877	0.52653		137.15		0.0
7	0.6096	0.08026		137.15		0.0
8	0.7112	0.07296		137.15		0.0
9	0.8128	0.07296		137.15		0.0
10	0.9144	0.07296		137.15		0.0
11	1.0160	0.06567		137.15		0.0
12	1.0973	0.07296		137.15		0.0
13	1.2192	0.04378		—		0.0
Wing						
1	0.0000	0.06487	0.1395 x 10 ⁻⁴	27.091	45.113	+0.04448 x 10 ⁻³
2	0.1016	0.12975	0.2790	27.091	45.113	+0.08896
3	0.2032	0.12975	0.2790	27.091	45.113	+0.13345
4	0.3048	0.12975	0.2790	27.091	45.113	+0.17793
5	0.4064	0.12975	0.2790	27.091	45.113	+0.22241
6	0.5080	0.12975	0.2790	27.091	45.113	+0.26689
7	0.6096	0.06487	0.1395	—	—	-0.13345
Tail						
1	0.0000	0.03243	0.06979 x 10 ⁻⁴	27.091	45.113	0.0
2	0.0508	0.06487	0.13958	27.091	45.113	0.0
3	0.1016	0.06487	0.13958	27.091	45.113	0.0
4	0.1524	0.06487	0.13958	27.091	45.113	0.0
5	0.2032	0.06487	0.13958	27.091	45.113	0.0
6	0.2540	0.03243	0.06979	—	—	0.0
^a Rotary inertia of lumped masses neglected						
^b Includes mass of stem and coil of shaker used in reference 1						
$k_1 = 350. \text{ N/m}, \quad k_2 = .0565 \text{ N-m/radian}, \quad k_3 = 87.6 \text{ N/m}$						

TABLE III. - CONSTRAINT EQUATIONS FOR AIRPLANE BEAM ASSEMBLY

1. $q_{27} - q_6 = 0$
2. $q_{34} \cos \theta_1 + q_{41} \sin \theta_1 = 0$
3. $-q_{34} \sin \theta_1 + q_{41} \cos \theta_1 + q_{19} = 0$
4. $q_{48} - q_{12} = 0$
5. $q_{54} \cos \theta_2 + q_{60} \sin \theta_2 = 0$
6. $-q_{54} \sin \theta_2 + q_{60} \cos \theta_2 + q_{25} = 0$

θ_1 = sweep of wing elastic axis = 30°

θ_2 = sweep of tail elastic axis = 30°

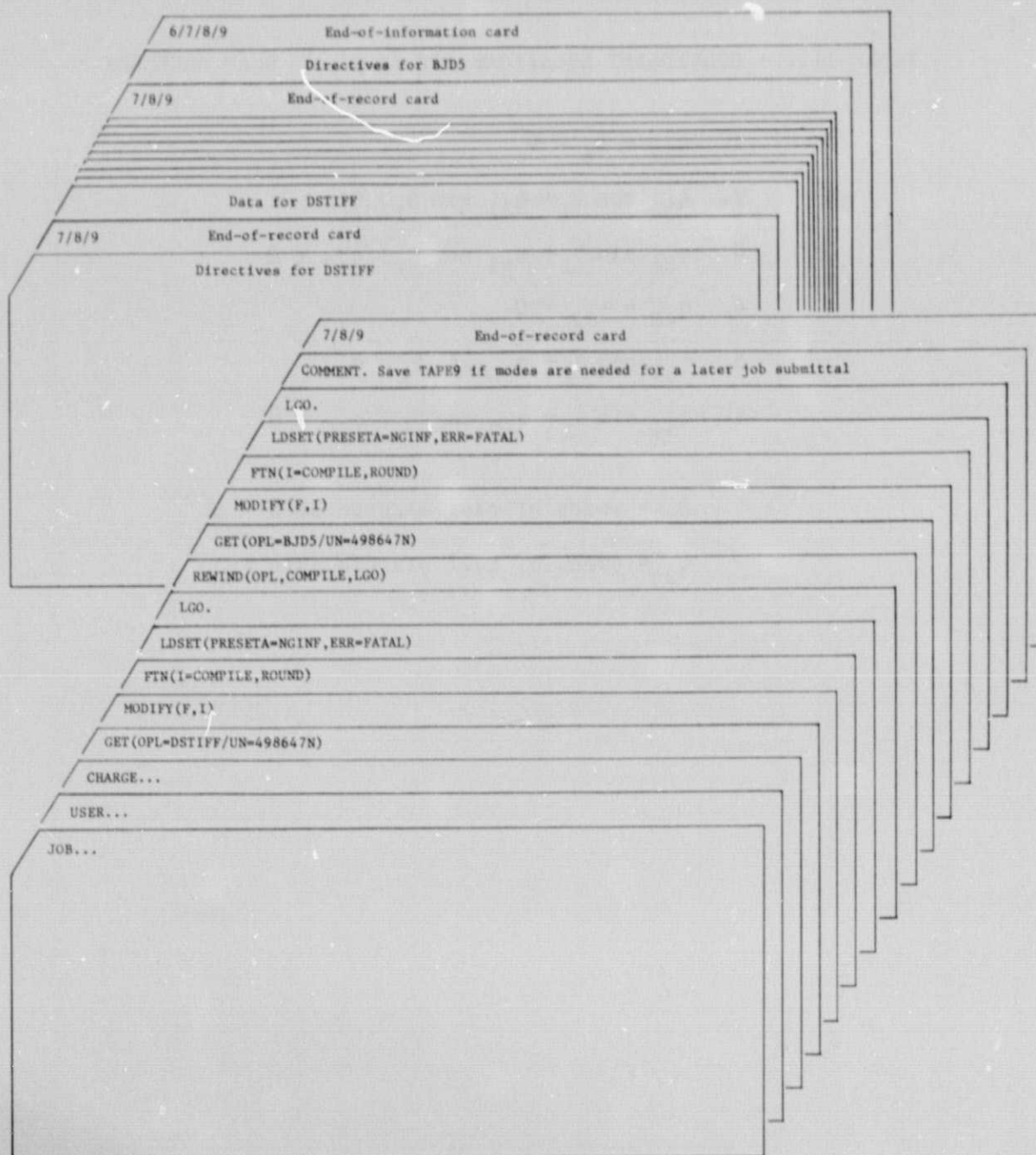


Figure 1.- Sample deck structure for execution of SUDAN program on CYBER 173/175 computers at the LaRC Computer Complex.

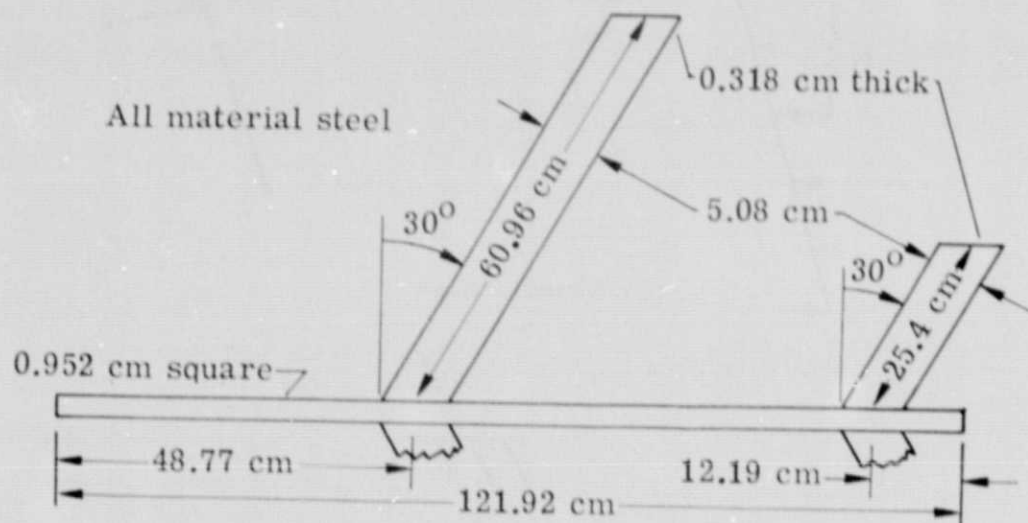


Figure 2.- Geometric properties of airplane beam assembly.

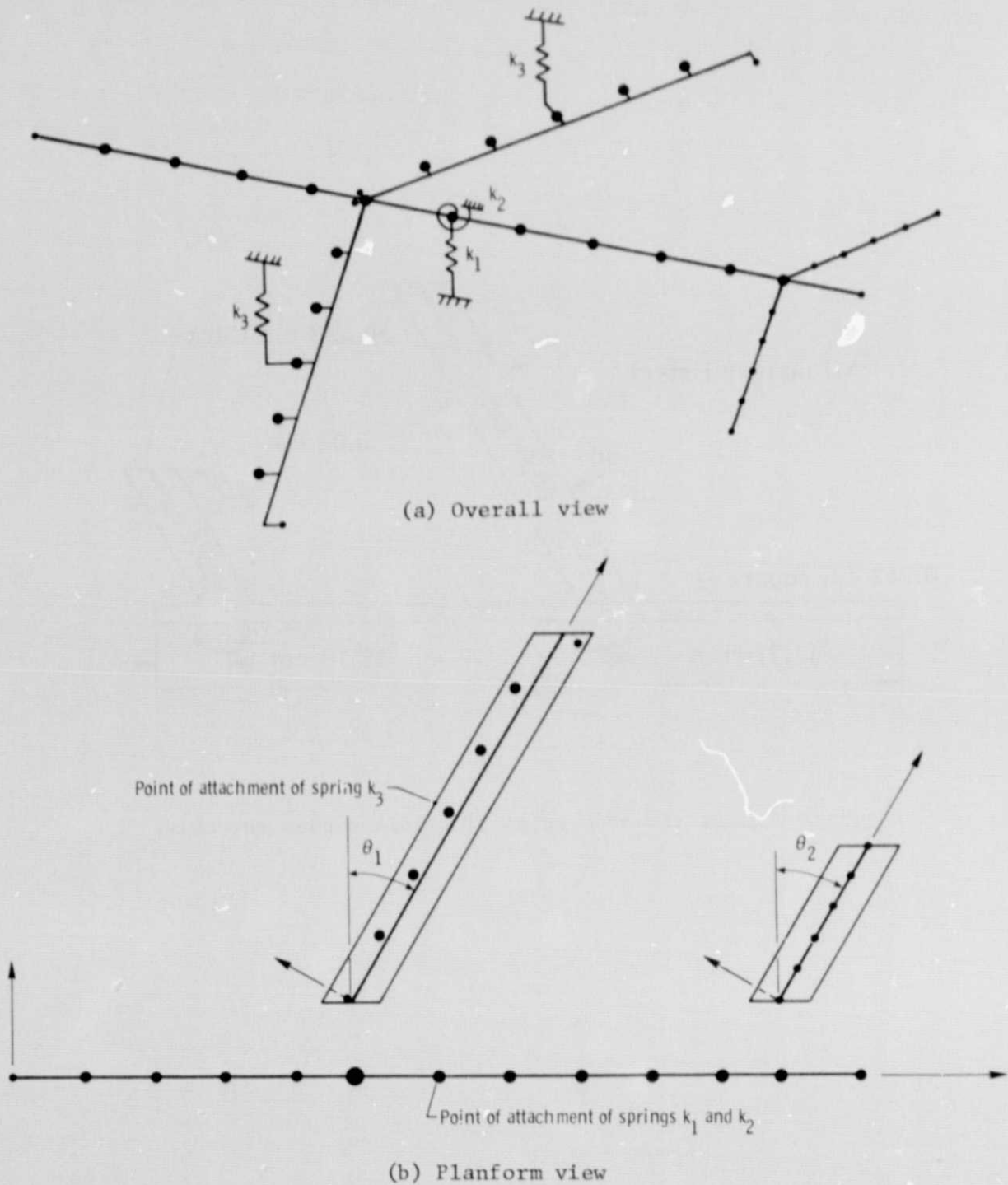


Figure 3.- Lumped-mass model of airplane beam assembly supported by springs.
 (Note: Offsets of masses and spring attachment points from wing elastic axis are exaggerated for illustrative purposes)